C = 3: Ternary Systems:
Example 1: Ternary Eutectic
Di - An - Fo

Note three binary eutectics
No solid solution
Ternary eutectic = M

As add components, becomes increasingly difficult to depict.
1-C: P - T diagrams easy
2-C: isobaric T-X, isothermal P-X...
3-C: ??
Still need T or P variable
Figure 7-2. Isobaric diagram illustrating the liquidus temperatures in the Di-An-Fo system at atmospheric pressure (0.1 MPa). After Bowen (1915), A. J. Sci., and Morse (1994), Basalts and Phase Diagrams.

P=1atm

X-X diagram with T contours (P constant)
Liquidus surface works like topographic map red lines are ternary cotectic troughs run from binary eutectics down T to ternary eutectic M
Separate fields labeled for liquidus phase in that field

Crystallization Relationships

Cool composition \( a \) from 2000\(^\circ\)C

At 2000\(^\circ\)C:
\[
\phi = ? \quad (1 \text{ liquid})
\]
\[
F = ?
\]
\[
F = C - f + 1 = 3 - 1 + 1 = 3
\]
\[
= T, \quad X(\text{An})\text{Liq}, \quad X(\text{Di})\text{Liq}, \quad \text{and} \quad X(\text{Fo})\text{Liq}
\]
only 2 of 3 X’s are independent

Next cool to 1700\(^\circ\)C intersect liquidus surface
What happens??
Pure Fo forms just as in binary

\[ \phi = ? \]

\[ F = ? \]
φ = 2 (Fo + Liq)
F = 3 - 2 + 1 = 2
If on liquidus, need to specify only 2 intensive variables to determine the system
T and \( X_{An}^{\text{liq}} \) or
\( X_{Di}^{\text{liq}} \) and \( X_{Fo}^{\text{liq}} \)
X of pure Fo is fixed
Continue to cool; Fo crystallizes and liquid loses Fo component. Xliq moves directly away from Fo corner “Liquid line of descent” is a -> b. Along this line liquid cools from 1700°C to about 1350°C with a continuous reaction: LiqA -> LiqB + Fo.
Lever principle → relative proportions of liquid & Fo

- **At 1500°C**
  - \( \text{Liq} \times + \text{Fo} = \text{bulk} \ a \)
  - \( \frac{\text{Liq} \times}{\text{Fo}} = \frac{a-\text{Fo}}{x-a} \)

At any point can use the lever principle to determine the relative proportions of liquid and Fo.
What happens next at 1350°C?
Pure diopside joins olivine + liquid
\( \phi = 3 \)
\( F = 3 - 3 + 1 = 1 \) (univariant at constant P)
\( X_{\text{liq}} = F(T) \) only
Liquid line of descent follows cotectic → M
- **New continuous reaction as liquid follows cotectic:**
  \[ \text{Liq}_A \rightarrow \text{Liq}_B + \text{Fo} + \text{Di} \]
- **Bulk solid extract**
- **Di/Fo in bulk solid extract using lever principle**

**Instantaneous ratio**

\[ \frac{\text{Fo-c}}{\text{Di-c}} \]
At 1300°C liquid = X

Imagine triangular plane X - Di - Fo balanced on bulk a

Liq x

Di

Fo m

Total amounts of three phases at any T can be determined by a modified lever principle:

\[ \frac{\text{Liq}}{\text{total solids}} = \frac{a-m}{\text{Liq-a}} \]

\[ \frac{\text{total Di/Fo}}{\text{cumulative amount}} = \frac{m-\text{Fo}}{\text{Di-m}} \]
At 1270 °C reach M the ternary eutectic anorthite joins liquid + forsterite + diopside
\[
\phi = 4, \quad F = 3 - 4 + 1 = 0
\]

**Discontinuous reaction:**
Liq = Di + An + Fo stay at 1270°C until consume liq

Below 1270 °C
have also solid
Fo + Di + An
\[
\phi = 3, \quad F = 3 - 3 + 1 = 1
\]
Fractional crystallization
Equilibrium melting
Partial Melting
Forsterite-diopside-anorthite (Fo-Di-An) diagram showing phases determined by low P (1 atm, Osborn & Tait, 1952) and high P (0.7 GPa, Presnall et al., 1978)
Figure 7-4. Isobaric diagram illustrating the cotectic and peritectic curves in the system forsterite-anorthite-silica at 0.1 MPa. After Anderson (1915) A. J. Sci., and Irvine (1975) CIW Yearb. 74.

3 binary systems:
- Fo-An eutectic
- An-SiO₂ eutectic
- Fo-SiO₂ peritectic

\[ c = \text{ternary peritectic} \]
\[ d = \text{ternary eutectic} \]
Begin with composition \( a \)

Fo crystallizes first

\( \text{phi} = 2 \)

\( F = 3 - 2 + 1 = 2 \)

\( \text{Xliq} \to b \) as cool

En now forms and \( F = 1 \)

\( \text{Xliq} \) then follows peritectic curve toward \( c \)
As Xliq follows peritectic can get bulk solid extract at any T by tangent method example at point x the tangent -> y as bulk solid

We know Fo, En, and Liq are the three phases since y = solids, it must be comprised of Fo and En but y falls outside the Fo-En join y = En + (-Fo) the reaction must be: Liq + Fo -> En which is a peritectic continuous reaction

If y fell between En and Fo it would be L = En + Fo
Continue cooling with continuous reaction until $X_{\text{liq}}$ reaches c at 1270°C.

Now get $\text{An} + \text{En} + \text{Fo} + \text{Liq}$

$\phi = 4$

$F = 3 - 4 + 1 = 0$

stay here with discontinuous reaction:

$Liq + \text{Fo} = \text{En} + \text{An}$

until Liquid used up

Since a plots in the $\text{Fo} - \text{En} - \text{An}$ triangle these must be the final phases.
Works the same way as the Fo - En - SiO₂ binary

If the bulk X is between Fo and En the liquid disappears first at the peritectic temperature and Fo + En remain as the final solids.

If, on the other hand, the bulk X lies to the right of En, then Fo is consumed first and the liquid continues to evolve toward the eutectic.
As a variation on this sequence: begin with $e$ 
Fo is first phase to crystallize. 
as it does so, $X_{\text{liq}} \rightarrow b$
where $E_{n}$ also forms 
as before get continuous reaction
$\text{Liq} + \text{Fo} = E_{n}$
but, when $X_{\text{liq}}$ reaches $f$ the $X_{\text{bulk}} (e)$ lies directly between $E_{n}$ and $X_{\text{liq}}$
Whenever $X_{bulk}$ lies directly between two phases, these two phases alone add to comprise the system: $En + f = e$

Thus $En$ and liquid are all it takes, so the olivine must be consumed by the reaction at this point.

$\phi = 2$ and $F = 3 - 2 + 1 = 2$

$X_{liq}$ then leaves the peritectic curve $\rightarrow En +$ Liq field (directly away from $En$)
Xliq $\rightarrow$ \textit{g} where \textbf{An} joins \textit{En} + Liq as before get \textbf{continuous} reaction  
\textit{Liq} = \textit{An} + \textit{En}  

Xliq $\rightarrow$ \textit{d} where \textbf{Tridymite} also forms again stay at \textit{d} with \textbf{discontinuous} reaction:  
\textit{Liq} = \textit{An} + \textit{En} + \textit{Sil} until last liq gone  
Since \textit{e} lies in the triangle \textit{En} - \textit{An} - \textit{SiO2}, that is the final mineral assemblage
Other areas are relatively simple

Liq $h$

Fo first and Xliq $i$
An joins and Xliq $c$
F = 0 and stay at $c$
until liquid consumed
never leave $c$
You pick others?
Note:

Binary character is usually maintained when a new component is added

- Eutectic behavior remains eutectic
- Peritectic behavior remains peritectic
- Solid solutions remain so as well
Figure 7-13. Pressure-temperature phase diagram for the melting of a Snake River (Idaho, USA) tholeiitic basalt under anhydrous conditions. After Thompson (1972). Carnegie Inst. Wash Yb. 71
Bowen’s Reaction Series

Discontinuous Series:
- olivine
- Mg pyroxene
- Mg-Ca pyroxene
- (Spinel)
- amphibole
- biotite

Continuous Series:
- Calcic plagioclase
- Calci-alkaline plagioclase
- alkali-calcic plagioclase
- alkaline plagioclase
- potash feldspar
- muscovite
- quartz

Temperature
The Effect of Pressure
The Effect of Water on Melting

Dry melting: $\text{solid} \rightarrow \text{liquid}$

Add water - water enters the melt

Reaction becomes:

$\text{solid} + \text{water} = \text{liq}_{(aq)}$

Figure 7-19. The effect of $\text{H}_2\text{O}$ saturation on the melting of albite, from the experiments by Burnham and Davis (1974). A J Sci 274, 902-940. The “dry” melting curve is from Boyd and England (1963). JGR 68, 311-323.
Figure 7-20. Experimentally determined melting intervals of gabbro under $\text{H}_2\text{O}$-free ("dry"), and $\text{H}_2\text{O}$-saturated conditions. After Lambert and Wyllie (1972). J. Geol., 80, 693-708.
Dry and water-saturated solids for some common rock types

More mafic rocks have higher melting point

All solids are greatly lowered by water

Figure 7-21. $H_2O$-saturated (solid) and $H_2O$-free (dashed) solids (beginning of melting) for granodiorite (Robertson and Wyllie, 1971), gabbro (Lambert and Wyllie, 1972) and peridotite ($H_2O$-saturated: Kushiro et al., 1968; dry: Ito and Kennedy, 1967).
We know the behavior of water-free and water-saturated melting by experiments, which are easy to control by performing them in dry and wet sealed vessels.

What about real rocks?

Some may be dry, some saturated, but most are more likely to be in between these extremes:

- a fixed water content $< $ saturation levels
- a fixed water activity